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Control of Orbital Drift of Geostationary Tethered Satellites

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This paper shows that the state of a dumbbell-type tethered geosynchronous satellite, in the neighborhood of a stable spoke equilibrium, is completely controllable by tether length in a central inverse squared gravity field in the subspace defined by a constant moment of momentum. The effectiveness of simple open-loop tether length changes on the adjustment of the system state is investigated, as is the usefulness of optimal control in orbit stabilization and orbit adjustment for both the linearized and full nonlinear models.

Introduction

THE dynamics of tethered satellite systems has been studied extensively during the last twenty years.[1-9] Due to recent missions, there has been a renewed interest both in the dynamics and in the control of tethered satellites. The first such experiment flown occurred in 1992 and consisted of a Shuttle-tethered subsatellite system in which tether lengths of approximately 20 km were planned. Shuttle-based subsatellites are also planned for the near future with a tether length of approximately 100 km. The difficulties associated with tether deployment and retrieval are well known in the literature. In the case of the manned Shuttle, the tether dynamics during retrieval are of major concern with respect to safety. These safety problems would be far less serious in the case of unmanned tethered satellite systems of the dumbbell type, which have been suggested for many years, and more recently again by Netzer and Kane.[10] The question of using attitude-orbit coupling for the control of such a simple tethered system is therefore finding renewed interest.

The present paper investigates attitude-orbit coupling, with particular attention being paid to the use of tether length control for satellite system repositioning. The use of tether deployment for satellite system relocation has been presented previously and is discussed in Ref. 11. The present paper goes beyond this, building on the work in Ref. 12, which showed that a dumbbell-type tethered satellite with tethers of negligible mass is completely controllable in the neighborhood of the spoke equilibrium in a subspace of the state space defined by the constant moment of momentum.

This result is reconsidered in the present paper. Under the same simplifying assumptions as in Ref. 12, simple control maneuvers in the neighborhood of the spoke equilibrium will be studied. In addition, a control law for orbit-attitude stabilization in the neighborhood of the spoke equilibrium will be tested, the control law being obtained via linear quadratic optimal control theory. If the orbit-attitude coupling is sufficiently strong it could possibly be used not only to stabilize the system's attitude but also to adjust the orbit. In particular, by means of a suitable tether control, a satellite in a geostationary orbit may be moved from one point of the orbit to another. Orbit-attitude coupling of tethered satellites therefore in principle permits the compensation of the in-plane drift of geostationary satellites without propellant expenditure. Since some of the present generation of geostationary satellites have their useful life limited by the total propellant available, tether control is studied in the present paper with respect to its viability, possibly permitting a longer useful life of satellites making use of this type of control. The energy necessary for tether control could possibly be obtained via solar radiation.

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Equations of Motion and Controllability

The system of two equal point masses and with a massless tether of length $2z$ moving in the plane of an inverse square force field is shown in Fig. 1. The Lagrangian associated with this system is

$$L = m/2\left[\dot{r}^2 + r^2\dot{\varphi}^2 + \dot{z}^2 + z^2(\dot{\varphi}-\dot{\theta})^2\right] + (m/2)\mu(1/r_1 + 1/r_2) \tag{1}$$

with

$$r_1^2 = r^2 + z^2 + 2zr\cos\theta \quad \text{and} \quad r_2^2 = r^2 + z^2 - 2rz\cos\theta \tag{2}$$

where the variables m, r, z, θ, and φ are defined in Fig. 1, μ is the constant of the gravitational field, and distances r, r_1, and r_2 all being measured with respect to the center of the Earth.

The generalized momenta corresponding to r, φ, θ are

$$p_r = \frac{\partial L}{\partial \dot{r}} = m\dot{r} \tag{3}$$

$$p_\varphi = \frac{\partial L}{\partial \dot{\varphi}} = m\left[r^2\dot{\varphi} + z^2(\dot{\varphi}-\dot{\theta})\right] \tag{4}$$

$$p_\theta = \frac{\partial L}{\partial \dot{\theta}} = -mz^2(\dot{\varphi}-\dot{\theta}) \tag{5}$$

and the Hamiltonian is

$$H = \frac{1}{2m}\left[p_r^2 + \frac{1}{r^2}(p_\varphi + p_\theta)^2 + \frac{1}{z^2}p_\theta^2\right] - \frac{m}{2}\dot{z}^2 - \frac{m}{2}\left(\frac{\mu}{r_1} - \frac{\mu}{r_2}\right) \tag{6}$$

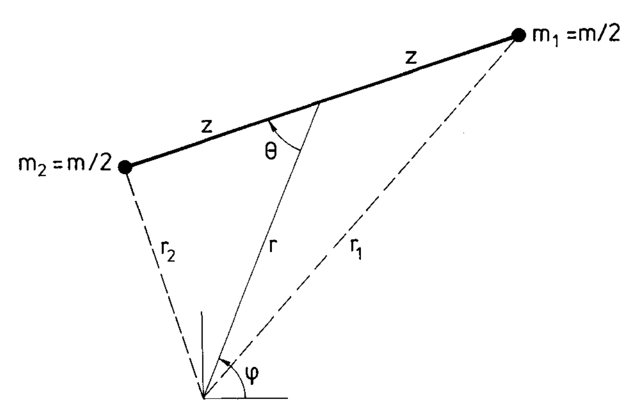

Fig. 1 System geometry.

The variable z is not transformed since z or $\dot{z}$ are used as control variables. Hamilton's equations are

$$\dot{r} = (1/m)p_r \tag{7}$$

$$\dot{\varphi} = \frac{1}{m}\frac{1}{r^2}(p_\varphi + p_\theta) \tag{8}$$

$$\dot{\theta} = \frac{1}{m}\left[\frac{1}{r^2}p_\varphi + \left(\frac{1}{r^2}+\frac{1}{z^2}\right)p_\theta\right] \tag{9}$$

$$\dot{p}_r = \frac{1}{m}\frac{1}{r^3}(p_\varphi + p_\theta)^2 - \frac{m}{2}\left[r\left(\frac{\mu}{r_1^3}+\frac{\mu}{r_2^3}\right) + z\cos\theta\left(\frac{\mu}{r_1^3}-\frac{\mu}{r_2^3}\right)\right] \tag{10}$$

$$\dot{p}_\varphi = 0, \qquad \dot{p}_\theta = \frac{m}{2}rz\sin\theta\left(\frac{\mu}{r_1^3}-\frac{\mu}{r_2^3}\right) \tag{11}$$

with the tension in the tether being given by

$$T = m\left[\frac{r}{2}\left(\frac{\mu}{r_1^3}+\frac{\mu}{r_2^3}\right)\cos\theta + \frac{z}{2}\left(\frac{\mu}{r_1^3}-\frac{\mu}{r_2^3}\right) - \ddot{z} + (\dot{\varphi}-\dot{\theta})^2 z\right] - \frac{\mu(r_1^2+z^2-r^2)}{2r_1^2 z} \tag{12}$$

The equations of motion admit the stationary solution corresponding to the spoke equilibrium

$$r(t) = r_0, \qquad \varphi(t) = \omega t, \qquad \theta(t) = 0, \qquad z(t) = z_0 \tag{13}$$

with $\omega^2 = (\mu/r_0^3)(1+\chi_0^2)/(1-\chi_0^2)^2$, and $\chi_0 = z_0/r_0$. In general χ_0 is small, so that the first two terms of the series $\omega^2 = \mu(1+3\chi_0^2 + \cdots)/r_0^3$ give a very good approximation.[13]

The dimensionless variables

$$\bar{r} = (r - r_0)/r_0 \tag{14}$$

$$\bar{\varphi} = \varphi - \omega t \tag{15}$$

$$\bar{\theta} = \theta \tag{16}$$

$$\bar{p}_r = p_r/(m r_0 \omega) \tag{17}$$

$$\bar{p}_\theta = (p_\theta + m z_0^2\omega)/(m z_0^2 \omega) \tag{18}$$

$$\bar{z} = (z - z_0)/z_0 \tag{19}$$

are introduced in Hamilton's equation, yielding

$$\bar{r}' = \bar{p}_r \tag{20}$$

$$\bar{\varphi}' = \left[1/(1+\bar{r})^2\right]\left[1+\chi_0^2\bar{p}_\theta\right] - 1 \tag{21}$$

$$\bar{\theta}' = \frac{1}{(1+\bar{r})^2}\left[1+\chi_0^2\bar{p}_\theta\right] + \frac{1}{(1+\bar{z})^2}\left[\bar{p}_\theta - 1\right] \tag{22}$$

$$\bar{p}_r' = \frac{1}{(1+\bar{r})^3}\left[1+\chi_0^2\bar{p}_\theta\right]^2 - \frac{1}{2}\eta_0^2\left[(1+\bar{r})\left(\frac{1}{\bar{r}_1^3}+\frac{1}{\bar{r}_2^3}\right) + \chi_0(1+\bar{z})\left(\frac{1}{\bar{r}_1^3}-\frac{1}{\bar{r}_2^3}\right)\cos\bar{\theta}\right] \tag{23}$$

$$\bar{p}_\theta' = \frac{1}{2\chi_0}(1+\bar{r})(1+\bar{z})\left(\frac{1}{\bar{r}_1^3}-\frac{1}{\bar{r}_2^3}\right)\sin\bar{\theta} \tag{24}$$

Linearizing Eqs. (21–24) in the nondimensional variables gives

$$\bar{r}' = \bar{p}_r \tag{25}$$

$$\bar{\varphi}' = -2\bar{r} + \chi_0^2\bar{p}_\theta \tag{26}$$

$$\bar{\theta}' = -2\bar{r} + (1+\chi_0^2)\bar{p}_\theta + 2\bar{z} \tag{27}$$

$$\bar{p}_r' = (2\alpha_0\eta_0^2 - 3)\bar{r} + 2\chi_0^2\bar{p}_\theta - 2\chi_0^2\beta_0\eta_0^2\bar{z} \tag{28}$$

$$\bar{p}_\theta' = -\beta_0\eta_0^2\bar{\theta} \tag{29}$$

where a prime indicates differentiation with respect to the new dimensionless time $\tau = \omega t$ and

$$\eta_0^2 = \frac{(1-\chi_0^2)^2}{(1+\chi_0^2)} = \frac{\mu/r_0^3}{\omega^2}, \quad \alpha_0 = \frac{1+3\chi_0^2}{(1-\chi_0^2)^3}, \quad \beta_0 = \frac{3+\chi_0^2}{(1-\chi_0^2)^3} \tag{30}$$

and

$$\bar{r}_{1,2}^2 = (1+\bar{r})^2 + \chi_0^2(1+\bar{z})^2 \pm 2\chi_0(1+\bar{r})(1+\bar{z}) \tag{31}$$

If the $\boldsymbol{x}$ is defined to be the state vector for the system, specifically, $\boldsymbol{x} = [\bar{r}, \bar{\varphi}, \bar{\theta}, \bar{p}_r, \bar{p}_\theta]^T$, then Eqs. (25–29) have the form

$$\dot{\boldsymbol{x}}(t) = \boldsymbol{A}\boldsymbol{x}(t) + \boldsymbol{B}u(t) \tag{32}$$

with

$$\boldsymbol{A} = \begin{bmatrix} 0 & 0 & 0 & 1 & 0 \\ -2 & 0 & 0 & 0 & \chi_0^2 \\ -2 & 0 & 0 & 0 & 1+\chi_0^2 \\ 2\alpha_0\eta_0^2 - 3 & 0 & 0 & 0 & 2\chi_0^2 \\ 0 & 0 & -\beta_0\eta_0^2 & 0 & 0 \end{bmatrix} \tag{33}$$

$$\boldsymbol{B}^T = 2(0, 0, 1, -\chi_0^2\beta_0\eta_0^2, 0) \tag{34}$$

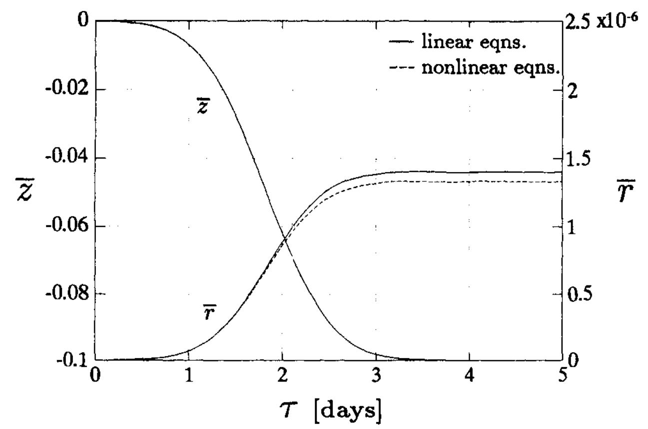

Fig. 2a Open-loop control input $\bar{z}$, response $\bar{r}$.

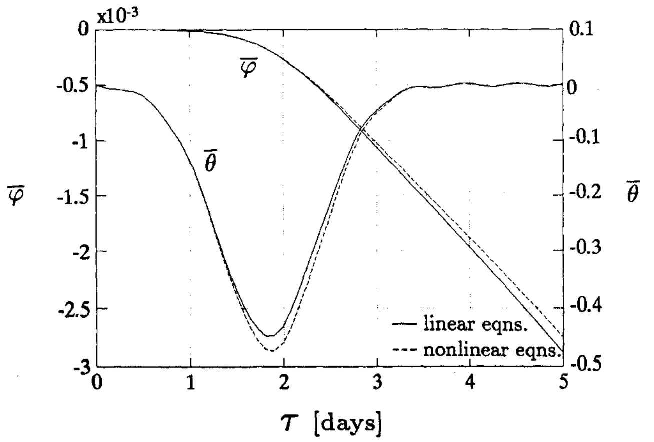

Fig. 2b Open-loop response $\bar{\varphi}$, $\bar{\theta}$.

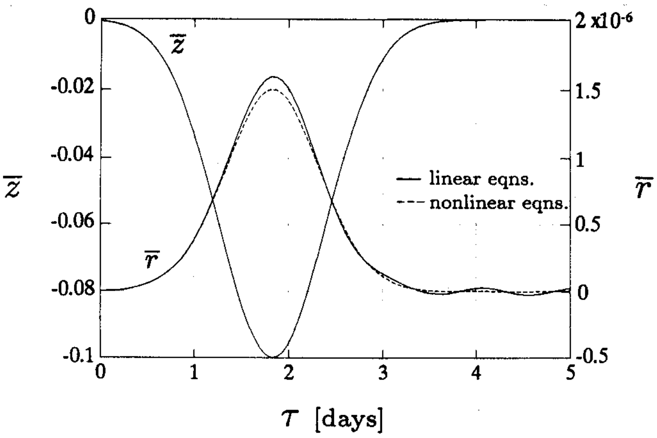

Fig. 3a Open-loop control input $\bar{z}$, response $\bar{r}$.

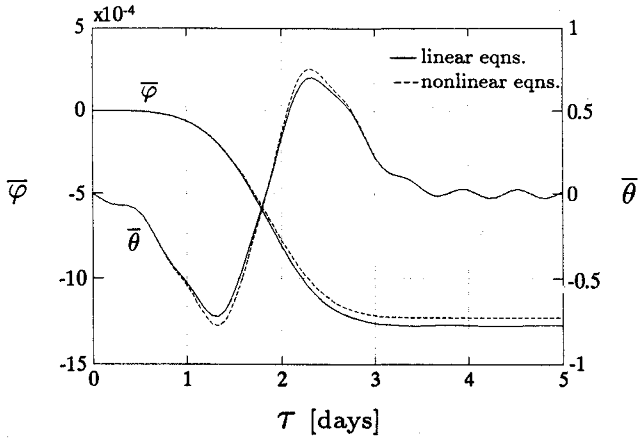

Fig. 3b Open-loop response $\bar{\varphi}$, $\bar{\theta}$.

and $u(t) = [z(t)]$. A somewhat tedious calculation gives the determinant of the Kalman matrix

$$\det(\boldsymbol{B}, \boldsymbol{AB}, \ldots, \boldsymbol{A}^4\boldsymbol{B}) = -2^9\left(\frac{3+\chi_0^2}{1-\chi_0^4}\right)^4\left(\frac{\chi_0^2}{1-\chi_0^2}\right)^3(\chi_0^2-9) \quad (35)$$

which is positive for $0<\chi_0<1$. Thus the Kalman matrix is of full rank, signifying that the system is completely controllable in the variables r, φ, θ, p_r, p_θ by means of the control variable $u = z$.

Simple Maneuvers Using Change of Tether Length

Simulation results for the open-loop response to prescribed changes in tether length indicate that this system's state may be easily moved from an initial state at one stable orbit to the vicinity of another. The changes in state variable $\bar{r}$ and/or $\bar{\varphi}$ may be accomplished an infinite number of ways, with examples shown in Figs. 2–4; however, each open-loop tether adjustment must be carefully tuned so that the maneuver is accomplished, leaving the system with a minimum error in the desired final state. All figures shown in this paper used a nominal tether length of $z = 100$ km and initial conditions for p_r and p_θ of zero.

The open-loop maneuvers tend to excite all modes of the system. This being done, it is very easy to tune the open-loop control function magnitude and duration so as to eliminate the response of the primary mode when the maneuver is accomplished. However, other modes are still active and in general it is difficult to select an open-loop control function that can move the system from an initial state to a desired final state and terminate all excited modes at the completion of the maneuver.

The use of tether length as a control variable is most effective in the stationkeeping adjustment of the orbit angle φ. This adjustment of φ through the control variable z is easily accomplished, and can return the other state variables and tether length to their previous premaneuver values with little added difficulty. This adjustment of φ, requiring only a short-term change in other state variables, is highly advantageous, allowing relatively easy adjustment from one equilibrium-state geosynchronous orbit to another. It is equally desirable that the maneuver be accomplished by returning the tether to its initial length, the underlying assumption being that the initial tether length was not arbitrary, but had a value that was selected so as to enhance mission performance. Consequently, long-term changes in tether length with magnitudes in excess of 10% simply for attitude adjustment purposes are considered undesirable.

The tether length and orbit radius are intimately related. The system can be moved only from one spoke equilibrium to another with identical moment of momentum. The relation between z and r for different spoke equilibria of the same angular momentum is given by the integral of moment of momentum $H = mr^2\dot{\varphi} + mz^2(\dot{\varphi}-\dot{\theta})$ with $\dot{\theta}=0$ and $\dot{\varphi}^2 = \mu(1+\chi_0^2)/r_0^3(1-\chi_0^2)^2$.

The attitude-orbit coupling increases with increased χ_0, consequently, when dealing with geosynchronous systems, as discussed here, the degree of coupling is light. Even with tether lengths of up to 200 km, the greatest foreseeable value of χ_0 is $<2.4\times10^{-3}$. As a result, significant changes in tether length are required to realize even modest changes in orbit radius.

Stabilization via Optimal Control

Because the system is completely controllable, control schemes such as pole placement, or optimal control via a linear quadratic regulator (LQR) may be used. In the case of pole placement, the closed-loop poles for a single input, single output system may be placed at any desired location using full state feedback and appropriate control gains. The algorithm

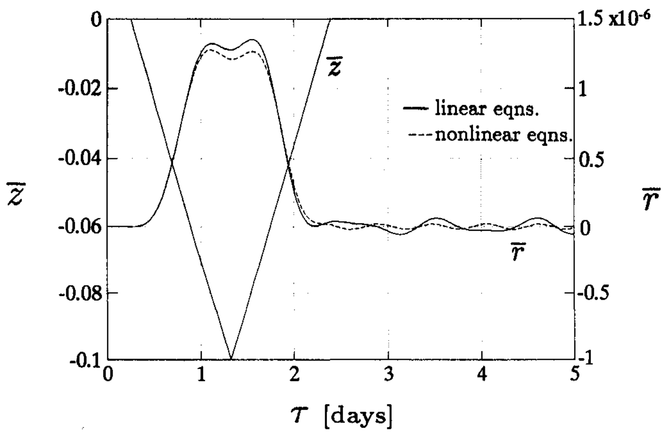

Fig. 4a Open-loop control input $\bar{z}$, response $\bar{r}$.

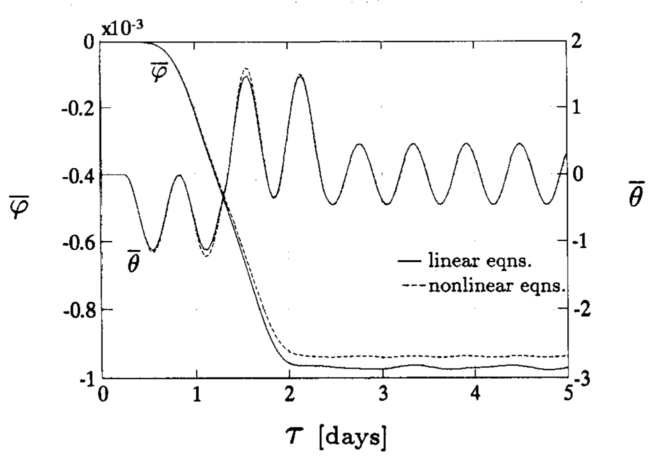

Fig. 4b Open-loop response $\bar{\varphi}$, $\bar{\theta}$.

used here in determining the necessary control gains associated with the desired pole locations is Ackermann's formula.[14] The problem with this method is that it is not usually clear where to place the poles for good response and satisfactory control activity. In general, the approach does not consider control effort, and unrealistic control gains are a common result. Furthermore, the method does not generalize well to multiple-input, multiple-output (MIMO) systems.[15] However, for the purposes of this paper, the approach lends itself well to demonstrating the control of system state through the use of tether length as a control variable.

The LQR, on the other hand, does consider control effort as well as state. This approach is especially useful for MIMO systems, since it produces gains that coordinate the multiple controls. The procedure synthesizes the control gain matrix $\boldsymbol{K}$ such that the feedback law

$$u = -\boldsymbol{K}x \tag{36}$$

minimizes the cost function

$$J = \int_0^\infty (X^TQX + U^TRU)\,\mathrm{d}t \tag{37}$$

where $\boldsymbol{Q}$ and $\boldsymbol{R}$ are 5×5 and 1×1 weighting matrices associated with the error in desired state and control effort, respectively.

Using the LQR approach, the system's attitude may be stabilized or the orbit parameters themselves may be adjusted, as shown in Figs. 5a and 5b. Due to the low level of attitude-orbit coupling which exists for geosynchronous orbits, the effectiveness in adjusting r is limited.

The gains determined by the LQR approach and the corresponding system response to errors are highly dependent on the weighting values chosen in this exercise. In all trade studies performed, the weighting matrix $\boldsymbol{Q}$ was given as $\boldsymbol{Q} = \mathrm{diag}[Q_{\bar r}, Q_{\bar\varphi}, Q_{\bar\theta}, Q_{p_r}, Q_{p_\theta}]$, with the results in Fig. 5 obtained with $\boldsymbol{Q} = \mathrm{diag}[10^0, 10^4, 10^{-4}, 10^4, 10^{-6}]$ and $\boldsymbol{R} = [10^{-3}]$, resulting in control gains $\boldsymbol{K} = [2\times10^4, 10^4, 0, 3\times10^{-4}, 3\times10^{-4}]$.

The system was found to have the greatest sensitivity to the relative magnitudes of the cost function weights R, Q_φ, and Q_{p_r}, and to a much lesser degree Q_r and Q_θ. As one would expect, virtually any system response characteristic may be produced by the appropriate selection of the weighting matrices element values. In effect, the system poles may be moved considerably if arguably unrealistic weighting values are used. However, significant control can be accomplished with what outwardly appear to be modest cost function weights and corresponding control gains.

In practically all maneuvers investigated, the force experienced by the tether remained positive (in tension). Only in extreme cases, involving exceedingly high control gains and/or values of $\bar z$ approaching -1 were negative tether forces encountered.

In general, the system behavior predicted by the linear and nonlinear models agreed well. This was particularly so for simulations that involved control gains determined through the LQR approach, and pole placement where the selected eigenvalues were real and less than -1. However, when real eigenvalues were selected on the order of -0.1 the behavior of the linear system and nonlinear system simulations differed markedly; Figs. 6a and 6b were generated with poles placed at -0.1, -0.9, -1.0, -1.1, and -1.2. In some more extreme cases the response predicted for the system as modeled, with the full nonlinear equations of motion and the linear control law, diverged completely (i.e., the nonlinear system was not stable with the control gains as determined for the stable linear

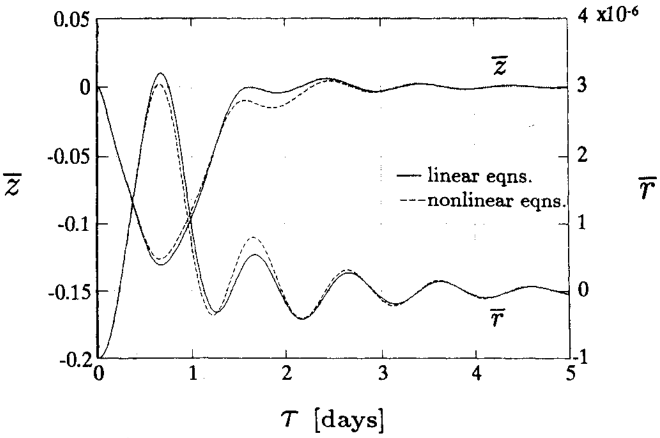

Fig. 5a LQR control input $\bar z$, response $\bar r$.

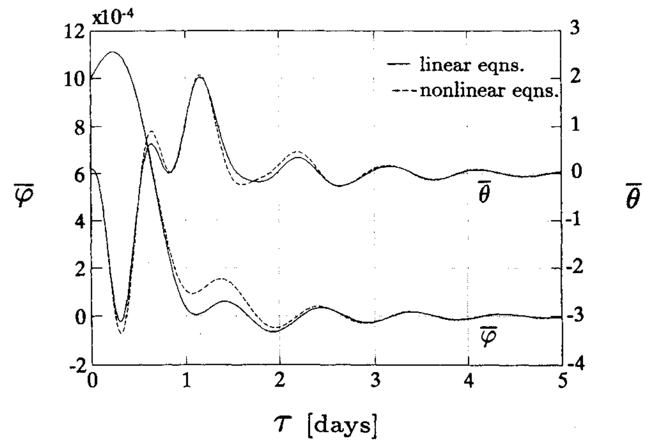

Fig. 5b LQR control response $\bar\varphi$, $\bar\theta$.

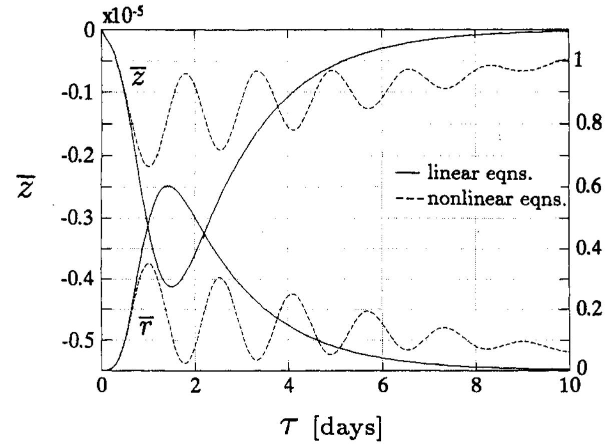

Fig. 6a Deviation in behavior between use of linear and nonlinear equations of motion, $\bar z$, $\bar r$.

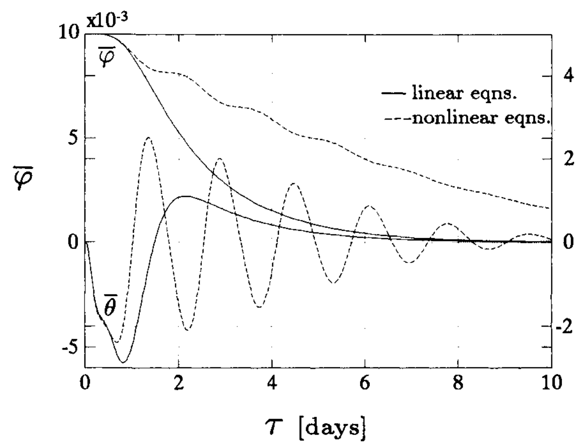

Fig. 6b Deviation in behavior between use of linear and nonlinear equations of motion, $\bar\varphi$, $\bar\theta$.

system) from the stable behavior predicted using the linearized equations of motion. Consequently, great care must be exercised when applying a control law determined from analysis of the linearized system to the more realistic, nonlinear one.

Superior performance could possibly be obtained using tether tension as the control variable, instead of z, and nonlinear control action in a time-optimal problem. However, the determination of necessary switching surfaces and the implementation of such control is difficult in systems with an order greater than two.[16] This is an area deserving of additional work.

Conclusions

It has been shown that the small planar motion of a system of two equal masses connected by an inextensible massless tether in a central inverse squared gravity field about its spoke equilibrium is completely controllable with respect to the tether length $2z$. Consequently, the use of tether length as a control variable allows for the correction of small orbit disturbances. This approach is ineffective in producing large changes in orbit radius r, however, it is quite effective in the adjustment of orbit angle φ.

Through full state feedback using control gains determined by the LQR approach, it was shown that tether length control can effectively stabilize a system's attitude and adjust the orbit. However, great variation in system performance is possible through the choice of cost-function weights, so additional attention needs to be paid to what realistic and probable weights would actually be.

Improved control performance might be realizable by using tether tension as the control variable and treating the system as a time-optimal problem.

Tether mass, elasticity, and damping could each play an important role in the dynamic behavior of the satellite system. The effects these parameters have on system controllability and behavior is the subject of ongoing work.

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